OCR
Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4768/01 Statistics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4768/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

1203667676

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** A stratified sample of pupils at secondary schools in a particular local authority is to be chosen in order to collect information on absenteeism. In the local authority there are 4 secondary schools, A, B, C and D, with 1310, 1453, 843 and 1110 pupils respectively.

(i) How many pupils should be chosen from each school in a stratified sample of 500 so that each school is represented proportionally? **[3]**

(ii) Suggest two possible criteria for stratification other than by school. **[2]**

(iii) State one advantage of choosing a stratified sample. **[1]**

(b) At a large secondary school, the median number of half days absent per pupil per year (based on several years' records) was known to be 23. Last year the school carried out a drive to lower the number of absences. A random sample of 12 pupils had been absent for the following numbers of half days during the year.

14 10 15 13 35 9 24 19 30 26 29 8

A Wilcoxon single sample test is to be carried out to see if the drive has been successful.

(i) Why might a Wilcoxon test be appropriate? **[1]**

(ii) What distributional assumption is needed for the test? **[1]**

(iii) Carry out the test, using a 5% level of significance. **[10]**

2 The distribution of the random variable X is thought to be well modelled by the following probability density function:

$$f(x) = \begin{cases} k(1+x) & \text{for } 0 \leqslant x < 5, \\ 0 & \text{elsewhere,} \end{cases}$$

where k is a positive constant.

(i) Find the value of k. **[3]**

(ii) Show that $P(a \leqslant X < a+1) = \frac{1}{35}(2a+3)$ for $0 \leqslant a \leqslant 4$. **[2]**

A random sample of 50 observations of X is summarised as follows.

x	$0 \leqslant x < 1$	$1 \leqslant x < 2$	$2 \leqslant x < 3$	$3 \leqslant x < 4$	$4 \leqslant x < 5$
Frequency	1	5	7	20	17

(iii) Test at the 10% level of significance whether the distribution of X is well modelled by $f(x)$. **[10]**

(iv) With reference to your calculations in part **(iii)** discuss briefly the outcome of the test. **[2]**

3 In agricultural research the oil content, as a percentage of the whole grain, of a cereal can be measured using near infra-red spectroscopy. An investigation into the effect of a particular treatment on the oil content of a certain cereal is being carried out. A sample of 10 plots of land is chosen and each plot is divided in half. In one half of each plot the cereal is grown with the treatment and in the other half the cereal is grown without the treatment. Subsequently the percentage oil content of the cereal for each half of each plot is measured and the results are as follows.

Plot	A	B	C	D	E	F	G	H	I	J
With treatment	41.1	44.3	42.4	48.2	52.4	54.6	35.9	33.6	51.1	47.0
Without treatment	42.5	37.7	42.1	32.4	42.7	41.5	36.9	31.7	52.6	41.2

A paired t test with a 5% level of significance is to be used to see if the treatment appears to make any difference to the mean percentage oil content of the cereal.

(i) Explain what is meant by a 5% level of significance in a hypothesis test. **[2]**

(ii) State the conditions necessary for the test to be carried out. **[3]**

(iii) Assuming the conditions stated in part **(ii)** are met, carry out the test. **[10]**

(iv) Find a 90% confidence interval for the population mean difference in the percentage oil content with and without the treatment. **[4]**

4 Paul has been trying a new route to work in the mornings. He collects a large random sample of times, in minutes, and calculates a 95% confidence interval for the population mean time by this route. The confidence interval is $(45.369, 47.231)$ and the sample variance is 20.3.

(i) Explain what is meant by a 95% confidence interval for a population mean. **[1]**

(ii) Calculate the sample mean and the sample size. **[4]**

Paul reverts to his usual route and the time, in minutes, to travel to work each morning is modelled by a random variable which is Normally distributed with mean 41.3 and variance 11.7. The time, in minutes, for Paul to travel home each evening is modelled by a random variable which is Normally distributed with mean 44.8 and variance 14.2. In the rest of this question all journeys are by Paul's usual route and may be assumed to be independent of each other.

(iii) Calculate the probability that, on a randomly chosen day, Paul's total travelling time will be less than 90 minutes. **[3]**

(iv) Calculate the probability that, on a randomly chosen day, the time for Paul to travel home will be more than 5 minutes longer than the time to travel to work. **[4]**

(v) Calculate the probability that, in a randomly chosen five-day week, the mean time for Paul to travel to work on Monday and Tuesday will be more than 3 minutes longer than his mean time to travel to work on Wednesday, Thursday and Friday. **[6]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

1(a)(i)	
1(a)(ii)	
1(a)(iii)	
1(b)(i)	
1(b)(ii)	

1(b)(iii)	
	(answer space continued on next page)

1(b)(iii)	**(continued)**
2 (i)	

2 (ii)	
2 (iii)	
	(answer space continued on next page)

2 (iii)	(continued)
2 (iv)	

3 (i)	
3 (ii)	
3 (iii)	
	(answer space continued on next page)

3 (iii)	(continued)

3 (iv)	
4 (i)	

4 (ii)	
4 (iii)	

4 (iv)	
4 (v)	
	(answer space continued on next page)

4 (v)	**(continued)**

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(a)	(i)	$\frac{500}{4716}\times\ldots$ = 138.88…, 154.05…, 89.37…, 117.68… = 139, 154, 89, 118	M1 A1 A1 **[3]**	Correct factor used for at least one school. All correct and given to at least 1 dp. FT any errors in the previous line provided that sum = 500.
1	(a)	(ii)	e.g. Sex (gender) Year group	B1 B1 **[2]**	Allow reasonable alternatives including ethnicity, birth date, distance from school
1	(a)	(iii)	e.g. Provides information on each stratum (as well as the population).	B1 **[1]**	Or representative
1	(b)	(i)	We have no information about the background population.	E1 **[1]**	o.e. Must include "population" o.e.
1	(b)	(ii)	Symmetry.	B1 **[1]**	
1	(b)	(iii)	H_0: m = 23 H_1: m < 23 where m is the population median number of days absent.	B1 B1	Both. Accept hypotheses in words, but must include "population". Do NOT allow symbols other than m unless clearly and explicitly stated to be a <u>population</u> <u>median</u>. Adequate definition of m to include "population".
			(table below)	M1 M1 A1	for subtracting 23. for ranks. ft if ranks wrong.

Absences	−23	Rank of |diff|
14	–9	7
10	–13	10
15	–8	6
13	–10	8
35	12	9
9	–14	11
24	1	1
19	–4	3
30	7	5
26	3	2
29	6	4
8	–15	12

Question			Answer	Marks	Guidance
			$W_+ = 1 + 2 + 4 + 5 + 9 = 21$	B1	($W_- = 3 + 6 + 7 + 8 + 10 + 11 + 12 = 57$)
			Refer to Wilcoxon single sample tables for $n = 12$.	M1	No ft from here if wrong.
			Lower 5% point is 17 (or upper is 61 if 57 used).	A1	i.e. a 1-tail test. No ft from here if wrong.
			Result is not significant.	A1	ft only c's test statistic. Dependent on all 3 M marks
			Insufficient evidence to suggest that the median number of days absent has been reduced.	A1	ft only c's test statistic. Dependent on all 3 M marks. Conclusion in context to include "on average" o.e.
				[10]	
2	**(i)**		Require $\int_0^5 k(1+x)\mathrm{d}x = 1$		
			$\int_0^5 k(1+x)\mathrm{d}x = k\left(x+\frac{x^2}{2}\right)\Big\vert_0^5 = k\left(5+\frac{25}{2}\right) - k\times 0 = \frac{35k}{2}$	M1 A1	Set up correct integral, including limits which may appear later. Allow method based on area, e.g., a trapezium. Integral correctly evaluated, or correct area obtained, in terms of k.
			$\therefore \frac{35k}{2} = 1 \qquad \therefore k = \frac{2}{35}$ Not 0.057….	A1	Set equal to 1 and rearranged for k.
				[3]	
2	**(ii)**		$\mathrm{P}(a \le X < a+1) = \int_a^{a+1} \frac{2}{35}(1+x)\mathrm{d}x$	M1	Set up correct integral, including limits which may appear later. Allow method based on area, e.g., a trapezium. Allow candidate's value of k.
			$= \frac{2}{35}\left((a+1) + \frac{(a+1)^2}{2} - a - \frac{a^2}{2}\right)$		
			$= \frac{2}{35}\left(1 + \frac{2a+1}{2}\right) = \frac{1}{35}(2a+3)$	A1	AG. Must be shown convincingly.
				[2]	
2	**(iii)**		H_0: The model is suitable / fits the data. H_1: The model is not suitable / does not fit the data.	B1	Both hypotheses. Must be the right way round. Do not accept "data fit model" oe.
			Expected frequencies are: $50 \times \left(\frac{3,\ 5,\ 7,\ 9,\ 11}{35}\right)$	M1	
			$= \left(\frac{30,\ 50,\ 70,\ 90,\ 110}{7}\right)$	A1	Accept either fractions or decimals.
			= 4.2857, 7.1428/9, 10, 12.8571, 15.7142/3		

Question			Answer	Marks	Guidance
			Merge first 2 cells: Obs f = 6, Exp f = 11.4285 $X^2 = 2.5786 + 0.9 + 3.9683 + 0.1052$ $= 7.552$ Refer to χ_3^2. Upper 10% point is 6.251. Significant. Sufficient evidence to suggest that the pdf of X is not well modelled by f(x)..	M1 M1 A1 M1 A1 A1 A1 **[10]**	Merge first 2 cells Calculation of X^2. Independent of previous mark. Awrt 7.55 Allow correct df (= cells – 1) from differently grouped table and ft. critical value only. Otherwise, no ft if wrong. No ft from here if wrong. P(X^2 > 7.552) = 0.0562. If cells not merged χ_4^2 10% point is 7.779; P(X^2 > 8.135) = 0.0867. ft only c's test statistic. ft only c's test statistic. Do not accept "data do not fit model" oe.
2	**(iv)**		e.g. The model overestimates for $0 \le x < 2$. The model underestimates for $3 \le x < 4$. "Large discrepancy" but no direction E1 max	E1 E1 **[2]**	Any 2 points relating to or explaining the outcome of the test. Other possibilities might include: The test would not have been significant at 5%. The sample is a bit small making it difficult to assess.
3	**(i)**		5% represents the probability of rejecting the null hypothesis … … when it is, in fact, true.	E1 E1 **[2]**	
3	**(ii)**		Must assume: Normality of population … … of <u>differences</u>. Sample is random.	B1 B1 B1 **[3]**	Ignore references to unknown variance and/or sample size.
3	**(iii)**		$H_0: \mu_D = 0$ $H_1: \mu_D \neq 0$ Where μ_D is the (population) mean difference in percentage oil content.	B1 B1	Both. Accept alternatives e.g. $\mu_A - \mu_B$ etc provided adequately defined. Hypotheses in words only must include "population". Do NOT allow "$\overline{X} = \ldots$" or similar unless $\overline{X}$ is clearly and explicitly stated to be a <u>population</u> mean. For adequate verbal definition. Allow absence of "population" if correct notation μ is used.

Question			Answer	Marks	Guidance
			<u>MUST</u> be PAIRED COMPARISON *t* test. Differences (with – without) are: –1.4 6.6 0.3 15.8 9.7 13.1 –1.0 1.9 –1.5 5.8 $\bar{x} = 4.93 \quad s_{n-1} = 6.310(4) \quad (s_{n-1}{}^2 = 39.822(3))$	 M1 A1	Allow "without – with" if consistent with alternatives for hypotheses above. Do not allow $s_n = 5.9886$ ($s_n{}^2 = 35.8401$).
			Test statistic is $\dfrac{4.93 - 0}{\frac{6.310}{\sqrt{10}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 0 + (c's 2.262) × $\dfrac{6.3104}{\sqrt{10}}$ (= 4.514) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's 2.262) × $\dfrac{6.3104}{\sqrt{10}}$ (= 0.416) for comparison with 0.)
			= 2.470(4).	A1	c.a.o. but ft from here in any case if wrong. Use of 10 – $\bar{x}$ scores M1A0, but ft.
			Refer to t_9. Double-tailed 5% point is ±2.262.	M1 A1	No ft from here if wrong. Must be minus 2.262 for "without – with" unless absolute values are being compared. No ft from here if wrong. P(|*t*| > 2.4704) = 0.03554.
			Significant.	A1	ft only c's test statistic as long as it includes their $s/\sqrt{n}$
			Sufficient evidence to suggest that the treatment appears to make a difference to the mean percentage oil content of the cereal.	A1 **[10]**	ft only c's test statistic as above Conclusion in context to include "on average" o.e.
3	**(iv)**		CI is given by 4.93 ±	M1	ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_9 is OK. Allow c's $\bar{x}$.
			1.833	B1	1.833 seen.
			× $\dfrac{6.3104}{\sqrt{10}}$	M1	Allow c's s_{n-1}.
			= 4.93 ± 3.6577= (1.271(9), 8.588(1))	A1 **[4]**	c.a.o. Must be expressed as an interval.

Question			Answer	Marks	Guidance
4	(i)		In repeated sampling, 95% of all confidence intervals constructed in this way will contain the true mean.	E1 **[1]**	
4	(ii)		Mean = (45.369 + 47.231)/2=46.3 $47.231 = 46.3 + \sqrt{20.3} \times 1.96 / \sqrt{n}$ $n = \frac{1.96^2 \times 20.3}{0.931^2} \approx 89.97(2) \qquad \therefore n = 90$	B1 B1 M1 A1 **[4]**	cao Sight of 1.96. Or equivalent. Must be an integer. FT candidate's mean.
4	(iii)		Time to work $X \sim \text{N}(41.3,\ 11.7)$ Time to home $Y \sim \text{N}(44.8,\ 14.2)$ $X + Y \sim \text{N}(86.1,\ \ 25.9)$ $\text{P}(X + Y < 90) = \Phi\left(\frac{90 - 86.1}{\sqrt{25.9}} = 0.7663\right) = 0.7783$	B1 B1 B1 **[3]**	Mean Variance cao
4	(iv)		Require $\text{P}(Y - X > 5)$ $Y - X \sim \text{N}(3.5,\ \ 25.9)$ $\text{P}(Y - X > 5) = 1 - \Phi\left(\frac{5 - 3.5}{\sqrt{25.9}} = 0.2947\right) = 1 - 0.6159 = 0.3841$	M1 B1 B1 A1 **[4]**	Allow equivalent alternatives, e.g. $Y > X + 5$ or $X - Y < -5$ Mean Variance cao
4	(v)		Require $\text{P}\left(\frac{X_1 + X_2}{2} > \frac{X_3 + X_4 + X_5}{3} + 3\right)$ Mean = 3(41.3 + 41.3) – 2(41.3 + 41.3 + 41.3) = 0 Variance = $\frac{1}{4}$ (11.7 + 11.7) + $\frac{1}{9}$(11.7 + 11.7 + 11.7) = 9.75	M1 A1 B1 M1 A1	For considering some $\overline{X}_2$ and $\overline{X}_3$ For $\overline{X}_2 - \overline{X}_3 > 3$ For 0 For1/4, 1/9, and 11.7 seen o.e. For 9.75

Question			Answer	Marks	Guidance	
			$P(Z > \frac{3-0}{\sqrt{9.75}}) = 1 - 0.8317 = 0.1683$	A1	cao	
				[6]		

4768 Statistics 3

General Comments

Candidates as a whole found this paper to their liking and scored well on all questions. As in previous years, candidates seemed to be far more comfortable carrying out calculations than with the other requirements of the paper such as producing hypotheses and conclusions, interpreting results and providing definitions. Many scripts suffered from a lack of precision which manifested itself in many ways: inadequate hypotheses; over-assertive conclusions; over-specified final answers yet too little accuracy carried forward in calculations; inaccurate reading of tables; and finally a large number of scripts were very difficult to read. Hypotheses and conclusions are awarded around 15% of the marks available on a paper and yet in many cases they do not receive an appropriate level of care and attention, sometimes appearing at the end of the question or obviously put in as an afterthought squeezed in between lines of working or at the side of the page.

Comments on Individual Questions

Question 1

Sampling and Wicoxon single sample test

This question was very well-done by most candidates and many scored full marks or close to it.

The Wilcoxon test is clearly well understood by most candidates.

Q1ai. was answered correctly by virtually all candidates. The handful who did not score full marks usually did not give integer answers or rounded incorrectly.

Q1aii. The most popular answers here, as expected, were age and gender. Other answers were allowed as long as they led to clear strata and used readily available data. So, for example, subjects studied was not allowed because it would not give clear strata, and parental income was not allowed because the data was not readily available.

Q1aiii. Here the most popular answer by far was that the sample would be representative, although a wide variety of terminology was used. A worthy minority gave the stronger response that stratified sampling also provided information about the individual strata.

Q1bi. A slim majority of candidates were successful here by stating either that there was no information about the background population or that the population was not known to be Normal. The most popular incorrect responses were that the median was known or that it was required to carry out a test on the median

Q1bii. Most candidates realised that the key word was symmetrical. Unfortunately, a minority attached it to ‘sample’ or ‘data’ or ‘the test’.

Q1biii. The great majority of candidates scored at least 8 marks here. If they lost any marks it was almost inevitably in the hypotheses or the conclusion. Most candidates used *m* to represent the population median number of days absent, but many did not use the word ‘population in their definition of *m,* or did not give any context. Some gave their hypotheses and failed to use the word population and thereby lost both marks for the hypotheses. The required conclusion was that there was insufficient evidence to suggest that the median number of days absent had reduced. A number of responses did not include the word median or contained no context. The statement “there is sufficient evidence that the median is still 23” is not equivalent. The test itself is well understood. A handful of candidates failed to subtract 23 and ranked the original data, or ranked actual values rather than absolute values, but these were rare.

Question 2 Continuous pdf and goodness of fit test

Almost all candidates were able to deal with the pdf at the beginning of the question. Goodness of fit tests are clearly well understood by candidates and around half of the candidates scored full marks.

Q2i. Almost all candidates obtained the correct value of k without difficulty

Q2ii. As the answer to this part was given, all candidates got the right answer. Most of them did so correctly. The most common errors seen were algebraic slips and the use of numerical values for the limits of the integral.

Q2iii. The test was done well by the great majority of candidates. The use of 'correlation' or 'relationship' in place of association' was much less common than in previous years in the hypotheses. Most candidates were able to calculate the expected values and the contributions correctly and a pleasingly high proportion of them also realised that the first two classes should be merged. The most common errors involved incorrect degrees of freedom and an occasional use of two-tailed values for the critical value.

Q2iv. 'High contributions because of the model overestimating the frequency for $0 < x < 2$, and underestimating for $3 < x < 4$' is the answer that was expected. A large number of candidates noted the high contributions but did not mention whether there was an under or over estimate. Others listed every class and stated whether there was an under or over estimate, but did not mention contributions. Others gave correct alternative suggestions including the fact that the conclusion of the test would have been different had the classes not been merged or if a 5% level of significance had been used.

Question 3 paired *t* test and confidence interval

Candidates found this question more challenging than the previous two questions. Although the paired *t* test is generally well understood, errors in the hypotheses and conclusions were not uncommon.

Q3i. Most candidates have clearly learned this.

Q3ii. Very few responses simply stated that the underlying population of differences should be Normal and that the sample should be random. Many candidates also included other possible requirements such as 'the sample size should be small' or that 'the variance should be unknown'.

Q3iii. The great majority of candidates knew how to carry out a paired *t* test. Errors in the hypotheses included the use of symbols other than μ, which were then not defined as the population mean, definitions which contained no context, and definitions which did not contain the word 'mean'. Some hypotheses had no definition of terms. Most, but by no means all, candidates were able to calculate $\bar{x}$ and s_{n-1} correctly, although a significant number used truncated values in what followed. The test statistic was usually correctly carried out, although on occasion $\sqrt{10}$ was missing. The majority of candidates then chose the correct critical value of *t,* although occasionally two tailed values were seen, as were values of *z.* The great majority of candidates made the correct decision in terms of rejecting the null hypothesis. A number of conclusions lacked context, some omitted the word 'mean' or were too assertive.

Q3iv. Most candidates know what is expected here, but there were a few errors that were not uncommon. These included the use of a *z* value rather than 1.833 or the use of an incorrect *t* value. A few candidates found a confidence interval for 'with treatment' or 'without treatment' or both. And also a few candidates gave their final interval to 5 or more significant figures.

Answer to 3iv (1.272, 8.588)

Question 4 Linear combinations of random variables

Candidates found this to be the most testing question. The last part of this question provided the opportunity for best candidates to produce some excellent work

Q4i. This part proved to be a good discriminator. Many candidates gave an accurate description of a 95% confidence interval, but a significant minority gave an explanation based on just one interval rather than the population of such intervals.

Q4ii. Although most candidates obtained the correct answers here, this did not seem to be familiar territory for candidates. Many solutions were unnecessarily convoluted. A large number of candidates clearly did not realise that $\bar{x}$ was simply the mid-point of the given interval. Common errors seen were the use of 1.645 instead of 1.96, the use of 20.3 instead of $\sqrt{(20.3)}$, and manipulation errors.

Q4iii. The great majority of the candidates answered this part correctly. The only errors seen were calculating the wrong tail and treating 11.7 and 14.2 as standard deviations rather than variances.

Q4iv. This part posed more difficulties for the weaker students. Some did not deal correctly with the difference between the two distributions being more than 5, and so some ended up in the wrong tail, some with a mean of 8.5 instead of 1.5 and some with an incorrect variance.

Q4v. This question effectively divided candidates into three groups. The first group did not work with means at all and were unable to make any progress. The second group worked with means and so found that the mean of the final distribution was zero. However this group were unable to calculate the variance of the final distribution. What was very common in this group was a lack of explanation as to how they obtained their variance. This lost them the opportunity to gain method marks. The third group, consisting of almost half of the candidates produced excellent, fully correct, solutions.

Answers 4ii $\bar{x}$ = 46.3, n = 90, 4iii. 0.7783, 4iv. 0.3841 4v. 0.1683

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GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	56	51	46	41	37	0
		UMS	100	80	70	60	50	40	0